So far, we have looked at equations where there is exactly one solution. Sometimes there are special cases when an equation may have many solutions or no solutions at all.

Let’s take a look at all three cases...

Solving Multi-Step Equations: Special Cases

One Solution
Many Solutions
No Solution
One Solution

- The final answer will result in the form “x = a” (the variable will equal SOMETHING).
- Only one real number can make the equation true.

\[2(x - 3) + 9 = 5 + x - 1\]

Infinitely Many Solutions

- The final answer will result in the form “a = a” (the same number will be on both sides of the equal sign).
- Any real number can make the equation true.

\[2(x + 10) - 17 = 5 + 2x - 2\]

No Solution

- The final answer will result in the form “a = b” (where a is a different number than b).
- There is no possible answer for this equation.

\[5x + 3 - 3x = 2(x + 3) - 5\]
Solving Equations: Special Cases - Guided Practice

Determine if each equation has one solution, many solutions, or no solution. If the equation has a solution, determine the solution to the equation.

1. \[2x + 2x + 2 = 4x + 2\]

2. \[3(x - 1) = 2x + 9\]
3. \[ 4(x + 3) - 4 = 8\left(\frac{1}{2}x + 1\right) \]

4. \[ \frac{3}{2}(2x + 6) = 3x + 9 \]

5. \[ 3 + \frac{3}{2}x + 4 = 4x - \frac{5}{2}x \]

6. Draw lines to match each linear equation to its correct number of solutions. How can you tell when an equation has no solution or many solutions?

- \[ 5(4 - x) = -5x + 20 \] \hspace{1cm} no solutions
- \[ -5(4 - x) = -5x + 20 \] \hspace{1cm} infinitely many solutions
- \[ 5(5 - x) = -5x + 20 \] \hspace{1cm} one solution